for the air-water system and gives better insight into rising bubble swarms than models based on the hindered settling of rigid particles. To check the general validity of the above model, systematic data for average bubble diameter, gas holdup, and flow regime as a function of gas and liquid velocities are needed.

ACKNOWLEDGMENT

The author wishes to express his gratitude to Dr. Norman Epstein for reading and criticizing the original manuscript. The continuing financial assistance from the National Research Council of Canada and the University of British Columbia is gratefully acknowledged.

NOTATION

D = diameter of tube, ft.
d_e = average bubble diameter, cm.
g = acceleration of gravity, ft./sec.^{2}
N_{Eo} = Eötvös number based on tube radius, $g\rho r^2/\sigma$
r = tube radius, ft.
r_e = equivalent bubble radius based on a sphere of equal volume, ft.
u_g = superficial gas velocity based on empty tube area, ft./sec.
u_L = superficial liquid velocity based on empty tube area, ft./sec.
V = rise velocity of single bubble in a tube, ft./sec.
V_o = energy destroying velocity of bubble swarm, ft./sec.
V_B = velocity of bubbles relative to wall for steady co-current gas-liquid flow, ft./sec.
V_∞ = rise velocity of single bubble in infinite media ft./sec.
V_s = slug rise velocity, ft./sec.
V_T = true liquid velocity as defined by Equation (10), ft./sec.

Greek Letters

ρ = density of liquid phase, lb./cu.ft.
σ = surface tension, poundals/ft.
λ = ratio of radii, r_e/r
ϵ = gas volume fractions
$\Delta\rho$ = difference between liquid and gas density, lb./cu.ft.

LITERATURE CITED

1. Ostergaard, K., and M. L. Michelsen, paper presented at AIChE meeting, Tampa, Fla. (May, 1968).
2. Hughmark, G. A., *Chem. Eng. Progr.*, **58**, 62 (1962).
3. Uno, S., and R. C. Kintner, *AIChE J.*, **2**, 3 (1956).
4. Harmathy, T. Z., *ibid.*, **6**, 281 (1960).
5. Maneri, C. C., and H. D. Mendelson, *ibid.*, **14**, 295 (1968).
6. Mendelson, H. D., *ibid.*, **13**, 250 (1967).
7. Marrucci, Giuseppe, *Ind. Eng. Chem. Fundamentals*, **4**, 224 (1964).
8. Happel J., and H. Brenner, "Low Reynolds Number Hydrodynamics," p. 387ff., Prentice-Hall, Englewood, Cliffs, N.J. (1965).
9. ———, and P. A. Ast, *Chem. Eng. Sci.*, **11**, 286 (1960).
10. Nicklin, D. J., *ibid.*, **17**, 693 (1962).
11. Dumitrescu, D. T., *Z. angew. Math U. Mech.*, **23**, 139 (1943).
12. Bridge, A. G., L. Lapidus, and J. C. Elgin, *AIChE J.*, **10**, 819 (1964).
13. Deraux, R., *Argonne Natl. Lab Trans.*, **61** (1964).
14. Levich, G. V., "Physical Hydrodynamics," p. 435ff., Prentice-Hall, Englewood Cliffs, N.J. (1962).
15. Davidson, J. F., and D. Harrison., *Chem. Eng. Sci.*, **21**, 731 (1966).
16. Dükler, A. E., Moyce Wicks, III, and R. G. Cleveland, *AIChE J.*, **10**, 38 (1964).
17. Baker, James L. L., and B. T. Chao, *ibid.*, **11**, 268 (1965).
18. Patrick, M., *Argonne Natl. Lab. Rept. 6581* (1962).
19. Ellis, J. E., and E. L. Jones, *Two Phase Flow Symposium*, Exeter, England (June 1965).
20. Towell, G. D., and C. P. Strand, Proc. Symposium Mixing, London, England, (June, 1965).

Latent Heat Estimation Using Altenburg's Quadratic Mean Radius

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An array of theoretical and semi-empirical relations is available (*1* to *3*) to predict latent heats of vaporization for pure substances. These methods as a rule use parameters associated with the vapor pressure data, critical data and molecular data (*4*), such as molecular weight and molecular volume. The difficulties in using the vapor pressure and critical point data are well known (*1*, *3*). On the other hand, methods based on the molecular weight and molecular volume (*4*) are not entirely satisfactory in the case of complex isomeric substances. It appears that a parameter describing molecular mass distribution as related to the configuration of the molecule, would provide a direct and useful way to correlate the latent heat of vaporization not only for the simple but also for these structurally branched (isomeric) compound series (for instance, alkane isomers).

Such a parameter is Altenburg's (*5* to *8*) quadratic mean radius $\overline{R_v^2}$ $\left(\overline{R_v^2} = \frac{1}{N^2}\sum_{i=1} n_i a_i^2\right.$, where n_i is the frequency of the distance a_i^2 occurring in the molecule and N is the number of the same kind atoms in the molecule$\left.\right)$. This parameter is associated with the mean moment of inertia of molecule (*9*) and so, connecting the molecular mass distribution with the particular geometry of the molecule permits to distinguish between the various isomers within the given compound class.

Using the published values of $\overline{R_v^2}$ (Table 1) and the available experimental values for the latent heat of vaporization as taken from Rossini (*10*),* the following correlating equation type was established:

$$\Delta H_{vb} = qX^b \tag{1}$$

where $X = \overline{R_v^2}/a_i^2$ and q and b are constants referring to a particular class of compounds. The conventional least squares treatment was used to evaluate the constants q, b

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* Tables 3, 4, 5, and 6 presenting computed [from Equation (4), using constants found in Table 2] and experimental values of ΔH_v for n-alkanes, cyclic alkane, and benzene compounds and alkane isomers; are deposited as document NAPS-00318 with the ASIS National Auxiliary Publications Service, c/o CCM Information Sciences, Inc., 22 W. 34th St., New York 10001 and may be obtained for $1.00 for microfiche or $3.00 for photocopies.

TABLE 1. USED ALTENBURG'S QUADRATIC MEAN RADIUS VALUES*

Compound	$\frac{\overline{R_v^2}}{a_i^2}$	Compound	$\frac{\overline{R_v^2}}{a_i^2}$
	n-Alkanes		
propane	0.5185	*n*-decane	2.6888
n-butane	0.8056	*n*-undecane	3.0134
n-pentane	1.1048	*n*-dodecane	3.3394
n-hexane	1.4132	*n*-tridecane	3.6665
n-heptane	1.7271	*n*-tetradecane	3.9943
n-octane	2.0449	*n*-pentadecane	4.3228
n-nonane	2.3657	*n*-hexadecane	4.6520
		n-heptadecane	4.9814
	Alkane Isomers		
i-butane	0.6875	3-methyloctane	2.1216
2-methylbutane	0.9511	4-methyloctane	2.0724
2,2-dimethylpropane	0.8000	3-ethylheptane	1.9743
2-methylpentane	1.2510	4-ethylheptane	1.9251
3-methylpentane	1.1965	2,2-dimethylheptane	1.9768
2,2-dimethylbutane	1.0370	2,3-dimethylheptane	1.9267
2,3-dimethylbutane	1.0895	2,4-dimethylheptane	1.9264
2-methylhexane	1.5672	2,5-dimethylheptane	1.9755
3-methylhexane	1.4865	2,6-dimethylheptane	2.0736
3-ethylpentane	1.4059	3,3-dimethylheptane	1.8300
2,2-dimethylpentane	1.3288	3,4-dimethylheptane	1.8288
2,3-dimethylpentane	1.3273	3,5-dimethylheptane	1.8776
2,4-dimethylpentane	1.4074	4,4-dimethylheptane	1.7809
3,3-dimethylpentane	1.2487	2-methyl-3-ethylhexane	1.7796
2,2,3-trimethylbutane	1.1701	2-methyl-4-ethylhexane	1.8284
2-methylheptane	1.8913	3-methyl-3-ethylhexane	1.6833
3-methylheptane	1.7984	3-methyl-4-ethylhexane	1.7308
4-methylheptane	1.7673	2,2,3-trimethylhexane	1.6845
3-ethylhexane	1.6744	2,2,4-trimethylhexane	1.7330
2,2-dimethylhexane	1.6464	2,2,5-trimethylhexane	1.8308
2,3-dimethylhexane	1.6142	2,3,3-trimethylhexane	1.6357
2,4-dimethylhexane	1.6449	2,3,4-trimethylhexane	1.6833
2,5-dimethylhexane	1.7377	2,3,5-trimethylhexane	1.7808
3,3-dimethylhexane	1.5230	2,4,4-trimethylhexane	1.6842
3,4-dimethylhexane	1.5525	3,3,4-trimethylhexane	1.5870
		3,3-diethylpentane	1.5857
2-methyl-3-ethylpentane	1.5214	2,2-dimethyl-3-ethylpentane	1.5366
3-methyl-3-ethylpentane	1.4306	2,3-dimethyl-3-ethylpentane	1.5382
2,2,3-trimethylpentane	1.4010	2,4-dimethyl-3-ethylpentane	1.6342
2,2,4-trimethylpentane	1.4931	2,2,3,3-tetramethylpentane	1.4431
2,3,3-trimethylpentane	1.3698	2,2,3,4-tetramethylpentane	1.5391
2,3,4-trimethylpentane	1.4612	2,2,4,4-tetramethylpentane	1.5885
2,2,3,3-tetramethylbutane	1.2500	2,3,3,4-tetramethylpentane	1.4906
2-methyloctane	2.2196		
	Cyclic Alkanes		
cyclopentane	0.7236	ethylcyclopentane	1.2930
methylcyclopentane	0.9788	cyclohexane	0.9167
		methylcyclohexane	1.2245
	Benzene Compounds		
benzene	0.8186	*n*-propylbenzene	1.9007
toluene	1.1459	*o*-xylene	1.3509
ethylbenzene	1.5227	*m*-xylene	1.4643
		p-xylene	1.5210

* These values are established by Altenburg (5 to 8).

TABLE 2. CONSTANTS FOR USE IN EQUATION (1)

Group	q	b	Avg. (Absolute) Deviation	Maximum Deviation
n-alkanes	5.935	0.467	0.8%	2.7%
cyclic alkanes	7.158	0.279	1.3%	−2.9%
benzenes	7.801	0.261	1.4%	4.1%
alkane isomers	6.331	0.482	3.5%	10.4%

in Equation (1). The calculated q and b values are listed in Table 2.

Equation (1) shows the maximum percentage deviation for alkane isomer series (Table 6, 2,2,3,3-Tetramethylpentane)† as 10.3%, while the average absolute error for the same compound class is estimated to be 3.5% (Table 2). However, except for 17 of the 66 compounds evaluated in this class, all have an error of less than 5% (Table 6). The best correlation is obtained for the *n*-alkane class: the maximum percentage deviation is found to be 2.7%, while the average absolute error is 0.8% (Table 3).‡

ACKNOWLEDGMENT

The financial assistance of the National Research Council of Canada is gratefully acknowledged.

NOTATION

ΔH_{vb} = latent heat of vaporization at normal boiling point (kcal./mole)
$\overline{R_v^2}$ = quadratic mean radius
b = constant
q = constant

LITERATURE CITED

1. Othmer, D. F., and Hwa-Nan Huang, *Ind. Eng. Chem.*, **57**, No. 10, 42 (1965).
2. Fishtine, S. H., *Hydrocarbon Process Petrol. Refin.*, **45**, No. 4, 173 (1966); Ind. Eng. Chem. Fundamentals, **2**, No. 2, 49 (1963); Ind. Eng. Chem., **55**, No. 21, 49 (1963).
3. Reid, R. C., and T. K. Sherwood, "The Properties of Gases and Liquids," Second Ed., McGraw-Hill, New York (1966).
4. Ibrahim, S. H., and N. R. Kuloor, *Chem. Eng.*, **73**, No. 25, 147 (1966).
5. Altenburg, K., *Z.-Chem.*, **4**, 357 (1964).
6. *Ibid.*, 398 (1964).
7. Altenburg, K., *Kolloid-Z*, **178**, 112 (1960).
8. *Ibid.*, **189**, 144 (1963).
9. Taylor, W. J., *J. Chem. Phys.*, **16**, 257 (1948).
10. Rossini, F. D., "Physical and Thermodynamic Properties of Hydrocarbons and Related Compounds," Carnegie Press, Pittsburgh, Penn. (1953).

APPENDIX

The percent deviation was calculated by the formula:

$$\% \text{ Deviation} = \frac{\text{Actual Value} - \text{Calculated Value}}{\text{Actual Value}} \times 100$$

where:

Actual Value = Experimental value found in literature

Calculated Value = Value from correlating equation

$$\text{Average Absolute } \% \text{ Deviation} = \frac{\Sigma\ \% \text{ Deviation}}{N_T}$$

where N_T = total number of compounds correlated

Maximum % Deviation = The maximum value of the % — Deviation found in the given compound class.

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‡ See footnote on page 469.